Homework Assignment 11
Due date: Monday Nov 24, 2008, midnight

Cooling of a dike filled with hot lava

Study the cooling of lava filled dike and derive the temperature distributions $T(x,t)$. The lava has an initial temperature of 1000°C, and host rock has one of 0°C. The dike has a width of $d=5$ meters. The heat capacity of the lava is 1.0 Joule/gramm/Kelvin. The density of the lava and the host rock is 2.7 gramm/cm$^3$. The heat conductance of both materials is 0.03 J/cm/seconds/Kelvin.

Discretize the dike into $N=50$ intervals. Then add a layer of host rock of width $2*d$ on each side. This determines your initial conditions. Choose Dirichlet boundary conditions.

**Q:** Main question of this problem: What is the temperature in the middle of the dike after (a) 1 hour, (b) one day, (c) one week, (d) one month, and (e) after one year?

(1) Write a Matlab code that solves the heat equation using the following scheme:

$$ T(x,t + \Delta t) = \frac{\Delta t}{\Delta x^2} \kappa \left[ T(x + \Delta x,t) - 2T(x,t) + T(x - \Delta x,t) \right] + T(x,t) $$

What unit of length do you use in your Matlab code? ______
What unit of time do you use? ______
What is the coefficient $k$ (lecture 19)? Specify value and units: ______
What is your grid spacing $\Delta x$? ______
What is your time step $\Delta t$? _____ (should be on the order of minutes)
Work out what the coefficient $\eta$ (see lecture 21)? ______
Exchange your answers to these questions with at least one member of your previously assigned team before you start coding!!!

Now write and run your Matlab code for simulation parameters you listed above, answer question Q, and enter your results in a table:

(2) Make a movie (see lab exercise) that shows the cooling process and upload it on bSpace it along with your Matlab code (continue with parts 3 and 4 on the next page)
(3) Increase the special resolution to N=100, 200, 400... points in the dike until the answers to Q change by less than 1%. Specify N and Δt. Enter your converged results in a second table here:

(4) Increase the width of the host rock layers to 4*d and 6*d until the results for Q are converged within 1%. Say what width was needed.