Computer Lab + Homework Assignment 12

Rössler attractor and Poincaré sections

This lab is about the Rössler attractor, a dynamical system that exhibits chaotic behavior similar to the Lorentz attractor. No MATLAB code will be provided because this exercise is similar to earlier labs and does not require much coding. Please modify your MATLAB routines from last week for the computation of the asteroid orbit and solve the following set of coupled ordinary differential equations:

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + a\,y$$

$$\frac{dz}{dt} = b + z\,(x - c)$$

x, y, and z change with time. The parameters a, b, and c are fixed during one simulation. Let us set c=2 for the moment. Obtain your personal values for a and b from your student identification (SID) number: a=b=0.1SID. So if your SID is 233445 then set a=b=0.1233445. Please keep using these values throughout this exercise. Please enter them here **a=b= . . . .**

Part I: Again, we want to use the 4th order Runge-Kutta method. Please adapt your file "kepler_ode.m" to solve the ODE given above. Then modify your version of "run_kepler_ode_euler05.m" and use x=0.1, y=0.2, and z=0.3 as initial conditions.

Now run your code for 200 units of time. Make and enter the following plots
   (a) x(t) versus t
   (b) 2D phase space plot showing x(t) versus y(t)
   (c) 3D phase space plot showing x(t), y(t), and z(t) (use `plot3(xx,yy,zz)`)
Now determine your time step by reducing it until you see no further change in the graphs above. Please be conservative and use a safe value for the rest of this exercise.

Enter your answers to the following questions
   (a) What kind of state did the system converge to?
   (b) How long did it approximately take to converge?

Now enter the 2D and 3D phase space plots again where you removed the initial, transient part and only show only the converged part. The converged parts should look much simpler than the full plots. If your time allows please consider running for 1000 units of time instead of 200 for the following exercises.

Part II: By changing the value of 'c', the Rössler attractor goes through a series of period doublings until it reaches a chaotic state. Please read at Wikipedia about it. Please change 'c' and enter the 2D and 3D phase space plots showing only the converged part for (Please specify c under each graph!):

(a) a period-2 orbit
(b) a period-4 orbit
(c) a period-8 orbit
(d) a chaotic orbit for low c,
(e) and a chaotic orbit for high c=20
(f) If you stumble across a period 3 or 5 orbit please enter it also but please waist no time looking for it!

Part III is about the Poincaré sections. Please make a plot showing x(t) versus z(t) only for value of t with y(t)=0. Enter one graph for a period-4 orbit showing only points after the system has converged. How many points do you expect to see?

Instructions: Please save your current MATLAB file under a new name and disable all current plots. Because we have a finite Δt, y(t) will never be exactly zero. So you have add an if statement that detects if y(t) and y(t+Δt) have opposite signs. Use the MATLAB sign(y) function. Only under this condition add a point x(t) versus z(t) to your plot. Enter you plot here:

Part IV Now we want to use Poincaré's method to make a bifurcation plot
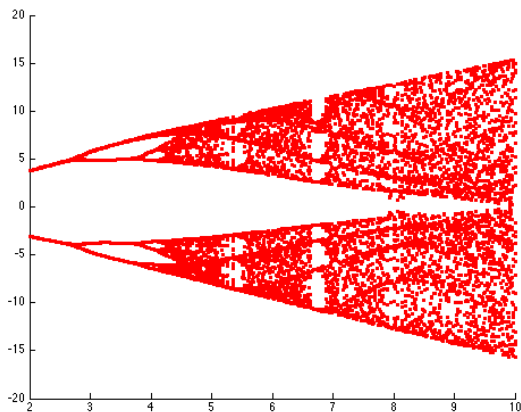
that shows the x(t) values (vertical axis) for all times t that satisfy y(t)=0. We start plotting points only after the system has converged. The ODE is solved repeatedly for many values of the parameter c that is shown on the horizontal axis. You need to add a loop to your code that allows run the ODE solver automatically for many values of 'c'. Just keep adding (c,x(t)) points to the plot if y(t)=0. Use 'hold on' and something like

```
plot(c,x,'s','MarkerSize',3,'MarkerFaceColor','b','MarkerEdgeColor','b');
```

Add the bifurcation plot for your values of a and b in the best 'c' resolution that you can afford here. The longer you run the individual ODE solver and the longer you wait for convergence the cleaner the plot will look. Submit the two MATLAB routines that you used for its generation along with your homework. If you got this far, well done!