

## Computer Lab Assignment 8

# 2D Heat Equation and Movies

The goal of this lab is to practice modeling cool things with the heat equation solver you have already used. We are not trying to model anything physical this week, so don't worry about units.

### Part 1: Model heat diffusing out of a pyramid-shaped initial temperature profile and make a movie of it.

- A) Check that your code for the 1D heat equation from part 2 of lab 7 works.
- B) Check that your code for the stationary state of the 2D heat equation from homework 6 works.
- C) Combine the two codes to model the real time heat diffusion in 2D.
  - i. Use the initial condition from homework 6 and the three nested “for” loops that step through time. Your 3 “for” loops probably look like this:

```

for iter=1:100
    Tnew=T; % just to copy value at the boundaries at 1 and N+1
    for i=2:N
        for j=2:N
            Tnew(i,j)= 0.25*(T(i-1,j)+T(i+1,j)+T(i,j+1)+T(i,j-1));
        end
    end
    T=Tnew;
    surf(X,Y,T)
    view([+35 154]);
    pause(0.01)
end
    
```

- ii. Here comes the critical step: Replace the calculation for Tnew(i,j) with the discretized 2D heat equation **in real time**. This is the 2D analog of the calculation for Tnew(i,j) from lab 7. The following slide from lecture 13 shows the right equation in math (not Matlab) language. Choose whatever you consider appropriate values of k, dt, dx and dy. We recommend setting dx = dy and choosing k and dt such that  $\eta = \frac{k\Delta t}{\Delta x^2} < \frac{1}{4}$ .

$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$

**Compare 1D and 2D**

$\frac{\partial T}{\partial t} = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$

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$T(x,t)$

$$T(x,t+\Delta t) = \frac{\Delta t k}{\Delta x^2} [T(x+\Delta x,t) - 2T(x,t) + T(x-\Delta x,t)] + T(x,t)$$

$T_i^n$

$$T_i^{n+1} = \eta [T_{i+1}^n - 2T_i^n + T_{i-1}^n] + T_i^n$$

$$\eta = \frac{\Delta t k}{\Delta x^2} < \frac{1}{2}$$

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$T(x,y,t)$

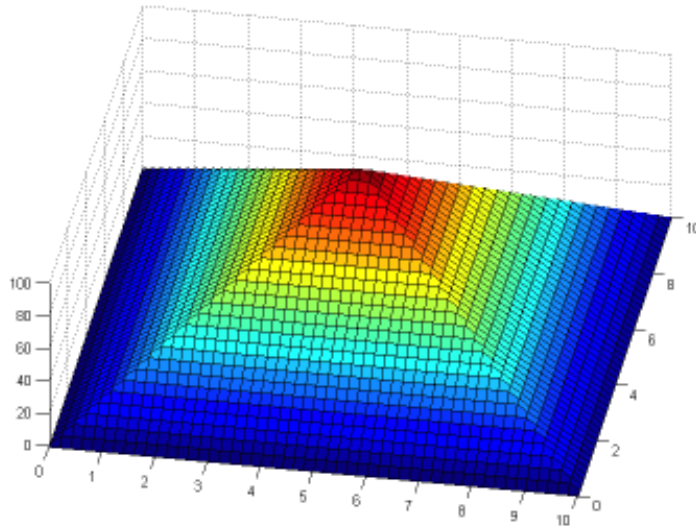
$$T(x,y,t+\Delta t) = \frac{\Delta t k}{\Delta x^2} [T(x+\Delta x,y,t) - 2T(x,y,t) + T(x-\Delta x,y,t)] + \frac{\Delta t k}{\Delta y^2} [T(x,y+\Delta y,t) - 2T(x,y,t) + T(x,y-\Delta y,t)] + T(x,y,t)$$

$T_{i,j}^n$

$$T_{i,j}^{n+1} = \eta [T_{i+1,j}^n + T_{i-1,j}^n + T_{i,j+1}^n + T_{i,j-1}^n - 4T_{i,j}^n] + T_{i,j}^n$$

$$\eta = \frac{\Delta t k}{\Delta x^2} < \frac{1}{4}$$

- D) Make the temperature of boundary 0 everywhere and make the initial temperature a pyramidal profile (this is the 2D analog of part 2 of lab 7) with a peak temperature of 100. Initial temperature profile should look like this:



You may use the function  $\min([f_1(x,y), f_2(x,y), f_3(x,y), f_4(x,y)])$ , where the four functions should be replaced by functions representing each face. Note that the  $\min()$  function requires a vector [...], not just four numbers.

- E) Make a movie by stringing together every the 1<sup>st</sup>, 11<sup>th</sup>, 21<sup>st</sup>, ... image, etc. Add the following to your code:

- i. Create a variable  $f = 0$  the beginning of your code. After setting  $T = T_{\text{new}}$  inside the outer loop, insert the following commands:

```
if mod(iter+9,10) == 0
    f=f+1;
    surf(X,Y,T)
    axis([0 L 0 L 0 100])
    saveas(gcf, ['heat_', sprintf('%03d', f), '.png'])
    pause(0.01)
end
```

- ii. You should now have a large number of individual frames in your Matlab directory. Please type in your Matlab prompt `pwd` to determine your Matlab directory name and `ls *.png` to double check that the files are there.

- iii. To convert the PNG frames into an animated GIF file, we need to use ImageMagick's program “convert”. It may be called “magick” on a PC:

**On a Mac**, you need to open a Terminal by clicking on the black square Terminal icon on you dock. (*This is what computers looked like before Microsoft came up with Windows OS by copying ideas from Apple's Mac OS who had copied it from some smaller company.*) Change into your Matlab directory with the command `cd` following by the directory from above.

```
cd ~/Documents/MATLAB
```

```
ls
```

If you do not see your PNG files, try this

```
cd /Users/\!tmfeps109/Documents/MATLAB
```

```
ls
```

(To make it hard, one needs to replace ! by \! If you directory name contains a space, you need to place a ! before it also.)

**On a PC**, launch the program “convert” or “magick” by double-clicking on the link on the desktop. Then change directory by typing in

```
cd "\Users!\!tmfeps109\Documents\MATLAB"
```

Type `dir` to see if the PNG file are there. If there is no functioning link on the desktop, launch the command window with “CMD” from the Windows search field.

- iv. Generate the animated GIF movie file by typing in either command:

```
convert -delay 20 heat_0???.png heatmovie.gif
```

```
magick -delay 20 heat_0???.png heatmovie.gif
```

This command tells the computer to take the images 01-99 named “heat\_0???.png” in the current directory, where ?? stand for any two characters, and put them into a single movie file called `heatmovie.gif`.

- v. The movie file `heatmovie.gif` will play back on most computers and is independent of Matlab. Watch it go on the lab computer by right-clicking on the file, then “open with” and select for example the Firefox, Safari, or even InternetExplorer.
- vi. You can convert more than 99 frames by changing 0?? to ??? but this may take a long time (~10 minutes) because the lab computers have rather little memory. You may try saving the PNG files in a lower resolution using the `print` rather than the `saveas` command (`-r` specifies the resolution in dots per inch):

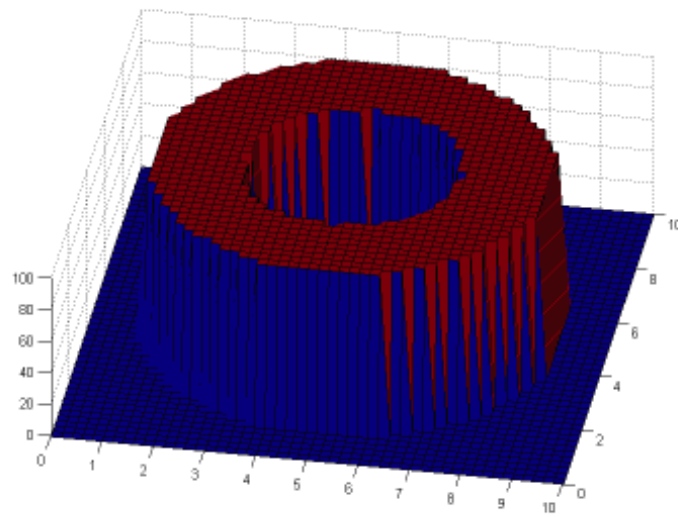
```
print(gcf, ['heat_', sprintf('%03d', f), '.png'], '-dpng', '-r100')
```

```
print(gcf, ['heat_', sprintf('%03d', f), '.png'], '-dpng', '-r50')
```

```
print(gcf, ['heat_', sprintf('%03d', f), '.png'], '-dpng', '-r20')
```

**Part 2: Model the heat diffusing out of an initial temperature distribution shaped like the letter “O” - 100 on a ring with your choice of width, and 0 both inside and outside the ring. Make a movie!**

Initial condition:



**Part 3: Model the same thing, but with a one-pixel size heat source in the very center of the “O”. Do this by adding 50 to the temperature of the very middle point after every diffusion time step. Make another movie of this.**