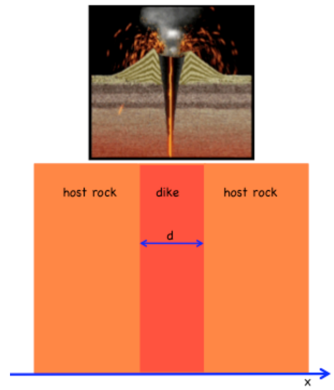
Student name: ***Enter your name here***

Homework 7: Cooling of a Lave Dike



This homework set is very much based on this week's computer lab. Here we study the cooling of lava filled dike and derive the temperature distributions *T(x,t)*. The lava has an initial temperature of 1250°C, and surrounding host rock host rock has one of 0°C. We assume the dike has a width of d=5 meters. For rock and lava, use a heat capacity of 1.15 Joule/gramm/Kelvin, a density of 2.72 gramm/cm^3, and a heat conductance of 0.025 J/cm/seconds/Kelvin.
Discretize the dike into N=50 intervals. Then add a layer of host rock of width 2×d on each side, which brings the total number of intervals to 5N. This determines your initial conditions. Choose Dirichlet boundary conditions, *T=0,* on each side.

Q: Main question of this problem: What is the temperature in the middle of the dike after (a) 1 hour, (b) one day, (c) one week, (d) one month, and (e) after one year?

(1) Write a Matlab code that solves the heat equation using the following scheme:

$$\boxed{T(x,t+\Delta t) = \frac{\Delta t}{\Delta x^2}\frac{\kappa}{c_P\rho}\left[T(x+\Delta x,t)-2T(x,t)+T(x-\Delta x,t)\right]+T(x,t)}$$

What unit of length do you use in your Matlab code? ______
What unit of time do you use? _____
What is the coefficient k (see lecture 10)? Specify value and units: ______
What is you grid spacing Δx? _____
What is your time step Δt? ____ (should be on the order of minutes)
Work out the coefficient η (see lecture 10)? ______
Exchange your answers to these questions with at least one student before you start coding or come to office hours!

Now write and run your Matlab code for simulation parameters you listed above, answer question Q, and enter your results for a)-e) in a table:

Cooling time	Temperature at dike center
1 hour	
1 day	
1 week	
1 month	
1 year	

(2) Increase the spatial resolution to N=100, 200, 300… points in the dike until all answers to Q change by less than 1.5°C. When you change N, please adjust Δt so that η remains constant. Enter your converged results in a second table here:

(3) Using again N=50, increase the width of the host rock layers from 2×d to 4×d, 6×d, 8×d, … until the results for Q are converged to a precision of 1.5°C. Specify the width that was needed and enter another table here:

(4) Finally compute how much heat the lava loses as function of time. The thermal energy is proportional to area under the $T(x,t)$ curve. Integrate the area inside the dike region and report what fraction is heat is lost for the time spans a)-e) by adding another column to the table in part (3).