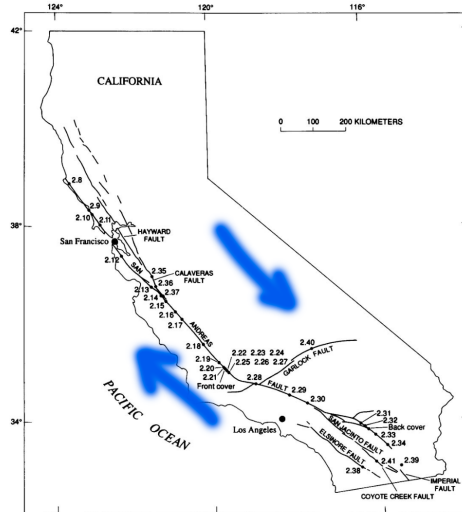


Student name: **Enter your name here**

## Homework Assignment 4

### (1) San Andreas Fault



Download the three data files “CA\_coast\_line.txt”, “CA\_coast\_line\_no\_islands.txt”, “plate.txt”, “SA\_fault.txt” and the Matlab code “san\_andreas.m” from bCourses. When you execute the latter file, it should bring up an image of the California coastline similar to the figure above. The San Andreas fault is shown in red. The land section to the West of it is shaded darker than the remaining land area. Everything is plotted in degrees of latitude and longitude.

The San Andreas fault marks the boundary of the Pacific plate, which moves Northwest at an angle of approximately  $32^\circ$  (counted from straight West, so that  $90^\circ$  would be straight North) at rate of 35 mm per year. This rate is measured relative to the motion of the North American plate (see arrows in the figure above or consult Wikipedia).

The irony is that part of the California belongs to the Pacific plate and slides Northwest with it. So we just have to wait a while until the Los Angeles airport (LAX) at  $33^\circ 56' 38.89''$  N and  $118^\circ 24' 09.04''$  W has moved conveniently close our campus.

a) Based on the provided information and the Earth radius, derive where the land mass West of the San Andreas fault has moved after 1, 5, and 25 millions years. (*When you convert from coordinates in degrees to distances, please ignore the curvature of the Earth and assume 1 degree of latitude corresponds to a distance of  $2\pi R_{Earth}/360$  and 1 degree of longitude is approximately equal to  $\pi R_{Earth}/180 \cdot \cos(35.5^\circ)$ .*) Assume the North American plate remains stationary. Determine the new location, update the coordinates in the variable ‘plate’ near the end of the code and replot the plate ‘patch’ one more time. Enter a new image for each of the three time intervals or combine all three in one image that you insert here:

b) How long do we have to until the latitude of the LAX airport coincides with that of campanile on our campus ( $37^\circ 52' 19.54''$  N and  $122^\circ 15' 28.19''$  W)? Answers with  $\pm 2\%$  will be accepted.

Everyone is free to *choose* between parts (2a) and (2b). Both will yield the same number of points but you will not earn more if you do both. Part (2a) is a simpler, (2b) a bit more advanced.

**(2a) Mandelbrot and Julia sets:** Both fractals use the same complex iteration formula:

$$z_{n+1} = (z_n)^2 + c \quad . \quad (\#)$$

For Mandelbrot set, you always start with  $z_0=0$  and  $c$  is the parameter that you vary. The Mandelbrot set is defined as set of all points  $c$  in the complex plane for which the series (#) does not diverge.

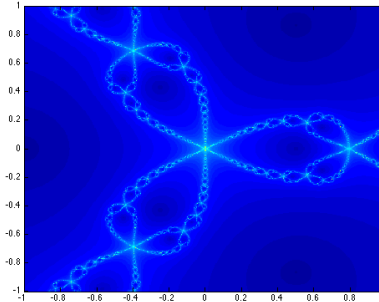
In case of the Julia set, the roles of  $z_0$  and  $c$  are reversed. You vary the starting number,  $z_0=x_0+iy_0$ . The parameter  $c$  stays fixed at some complex number to be specified beforehand. The Julia set is defined as the set of complex points  $z_0$ , for which the series (#) does not diverge. Different Julia set pictures emerge for different values of  $c$ .

- Now start from the latest version of Mandelbrot program that you modified in this week's computer lab. Reverse the roles of  $c$  and  $z_0$ . Fix  $c$  at  $-0.4+I*0.6$ . Investigate the interval  $-3<x_0<+3$  and  $-3<y_0<+3$  and locate the fractal region. Zoom in a little bit so that the fractal region fills the picture. Set  $Z(j,i)=n$  as explained in the lab handout. Insert your image in decent resolution here:
- Now zoom in deeper and deeper again until the  $x_0$  and  $y_0$  interval is less than 0.001 in size. Insert your best image here and provide your  $x_0$  and  $y_0$  ranges. You may have to increase the maximum number of iterations. Please keep  $c$  at  $-0.4+I*0.6$  fixed.
- Now zoom out again and change  $c$  to  $-0.85+I*0.22$ . Insert your picture showing the fractal region here:
- Now explore the  $c$  parameter region on your own. Zoom in until the interval size is less than 0.001. Insert your most beautiful picture here. Please specify  $c$  and the  $z_0$  interval.

**(2b) Newton fractals:** Believe it or not but Newton's method works for complex numbers also. It finds the zeros of a complex function  $f(z)$ . For this assignment please derive the iteration formula (see lecture on root finding) for the function  $f(z)=z^3+1$ :

$$Z_{\text{new}} = Z - \dots$$

(1) Enter this formula and the function 'f' in the code below. There is no constant 'c' in this problem. Instead the points x and y define the initial point,  $z_0=x+iy$ , where the algorithm starts to look for zeros. You should find an image like this:



(2) How many zeros does the function  $f(z)=z^3+1$  have in the complex plane? Describe where they are located in the picture. **Give a brief explanation of the colors nearby.**

(3) Change the code below for the function  $f(z)=z^n+1$  for  $n=4, 5,$  and  $6$ . **Enter the three resulting images.**

```
clf ; clear
xmin = -0.99999;
xmax = +1.0;
ymin = -1.0;
ymax = +1.0;
nPoints = 401;
fstop = 1e-06;

for i=1:nPoints
    for j=1:nPoints
        x(i) = ...
        y(j) = ...
        z = complex(x(i),y(j)); % this converts (x,y) to a complex number 'z'

        it_max = 200;
        for n = 1:it_max
            f = ...
            if (abs(f)<fstop)
                break;
            end
            z = z - ...
        end

        %store the results in the matrix ZZ(nPoints,nPoints)
        if (n==it_max) ZZ(j,i)=0; % make it '0' for blue if |z| is small and series has not diverged
        else ZZ(j,i)=n; % make it '1' for red if |z| is large and series has diverged
        end

    end
    fprintf('i= %d of %d points in X direction: x= %f\n',i,nPoints,x(i));
end

imagesc(x,y,ZZ); % plot the image
set(gca, 'YDir', 'normal');
```